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Survey of Nongravitational Forces and Space Environmental Torques: Applied to the Galileo

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A detailed survey of nongravitational forces and space environmental torques acting upon the Galileo spacecraft during its interplanetary flight to Jupiter is given. It includes simple analytic equations to model the first-order effect of solar, planetary, and spacecraft radiation; solar wind; meteoroids; cosmic rays; magnetic fields; Newtonian drag forces; and leakage of the propulsion system. The model parameters are taken from recent spaceflight data. The result is a model of the magnitudes of the disturbing forces and torques. It provides a useful tool for the analysis of the Galileo and future spaceflight missions.

Introduction

ON October 18, 1989, the Galileo spacecraft was launched on an interplanetary trajectory to Jupiter in order to study the Galilean moons, the Jovian environment and atmosphere, and the asteroids Gaspra and Ida. The flight path is illustrated in Fig. 1. During the mission the spacecraft will be exposed to a variety of space environmental forces and torques, which will directly or indirectly influence the spacecraft motion. Because of the high performance required by science experiments, a detailed survey of all possible nongravitational forces and space environmental torques along with an analysis of their effects on the spacecraft is necessary. To provide a profile of the magnitude of the forces and torques with respect to the spacecraft during the mission, the calculations were performed for five different reference points: near Earth (1 AU + $10R_E$), second Earth flyby (1 AU + $1.05R_E$), near Venus (0.72 AU + $3.75R_V$), interplanetary (3 AU), and near Jupiter (5.2 AU + $4R_J$).

Galileo is one of the first interplanetary dual-spin spacecraft. Its cruise configuration is shown in Fig. 2. The high-gain antenna (HGA), the radioisotope thermoelectric generators (RTGs), the large magnetometer science boom, and the retropropulsion module (RPM) are all located on the rotor which spins at 3.15 rpm, whereas the scan platform and probe (which will enter the Jovian atmosphere) are located on the stator, which remains inertially fixed.

Because of the stringent scientific and navigational requirements (which include the capability of accurately mapping the gravitational fields of Jupiter and its satellites and the exciting possibility of discovering gravity waves), the "3σ" uncertainties in the knowledge of the average acceleration (the difference between the modeled and unmodeled accelerations) must not exceed 3×10^{-12} km/s² (3σ). From launch to the arrival at Jupiter, the mass of the Galileo spacecraft varies between 2550–1370 kg, resulting in mission requirements equivalent to 3σ force uncertainties ranging from 8×10^{-6} to 4×10^{-6} N. (Throughout this paper, the terms "3σ" and "better than 99.73% probability" are used synonymously. This is equivalent to assuming that all error sources or the combined effect of the error sources have a Gaussian distribution.)

In this paper a general approach is taken in the analysis of the forces and torques on the Galileo spacecraft so that the results can be useful in the analysis of other spacecraft. An attempt is made to survey all sensible forces and torques and to numerically estimate their order of magnitudes by simple analytic formulas. The forces considered include those arising

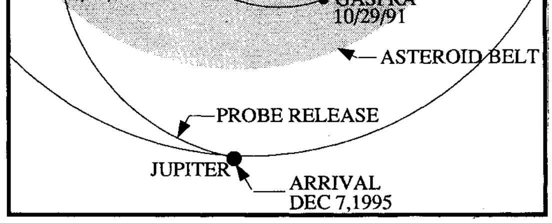

Fig. 1 Galileo trajectory.

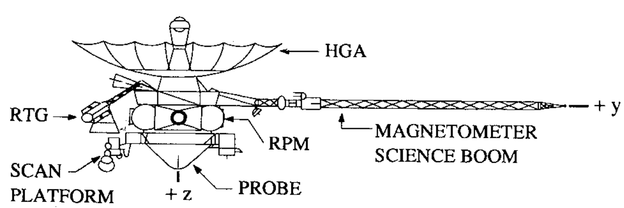

Fig. 2 Cruise configuration of the Galileo spacecraft.

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from 1) electromagnetic radiation (Sun, Earth, Venus, Jupiter, and spacecraft), 2) particle collisions (solar wind, meteoroids, cosmic rays, and atmospheres), 3) magnetic fields (Sun, Earth, Venus, and Jupiter), and 4) nonpropulsive mass expulsion. The torques considered include those due to the above forces as well as gravity gradient torques from the Sun, Earth, Venus, and Jupiter. The simplest equations are reported for each source in the interest of brevity (books and lengthy reports have been written about practically each item).

The forces ignored in this study include gravity (Newtonian and relativistic) which is modeled carefully in the trajectory analysis and forces from intentional spacecraft propulsive maneuvers, since we are interested in perturbations during the coasting phases of the trajectory. The torques ignored include all nonenvironmental torques induced by spacecraft moving parts such as fuel slosh, damping and structural flexing, and torques caused by spacecraft propulsive maneuvers. The only exception is the unintentional spacecraft mass expulsion torque due to gas leakage of the propulsion system. This is a well known effect which is considered too important to ignore.

In the analysis which follows, all of the forces are calculated in Newtons and all torques in Newton-meters.

Electromagnetic Radiation Forces

When electromagnetic radiation strikes a surface, there is a reaction force (similar to that from particle collisions) that is proportional to the area of the surface and the momentum flux. To precisely determine the force, the momentum flux incident on the surface and the reflected flux must be known. Edwards and Bevans[1] have shown that the reflected flux can be determined analytically from the reflection distribution function and the directional emissivity. In practice, these surface properties are not usually well known. Instead, the electromagnetic radiation forces are modeled by assuming that the incident radiation is either specularly reflected, absorbed, diffusely reflected, or some combination of these.[2] Figure 3 indicates the basic models. Sophisticated models can be built up for complex geometric shapes to obtain the electromagnetic force. These details are beyond the scope of this paper, but the methods are well documented.[3]

For our purposes, the magnitude of the force acting upon the spacecraft due to external electromagnetic radiation is given by

$$F = k_{\mathrm{elm}} A f_{\mathrm{elm}} / c \tag{1}$$

where k_{elm} = a dimensionless constant such that $k_{\mathrm{elm}} < 1$ for translucent material, $k_{\mathrm{elm}} = 1$ for a perfect blackbody, and $k_{\mathrm{elm}} = 2$ for a specularly reflective body; and f_{elm} = mean integrated energy flux (W/m^2), c = speed of light (m/s), and A = effective spacecraft surface area (m^2). (See Table 1 for Galileo spacecraft.)

The factors k_{elm} and A can be adjusted to obtain the best effective values that model the overall radiation force.

Equation (1) can now be used in the discussion and evaluation of forces from solar radiation, reflected solar radiation, planet thermal radiation, and radio waves.

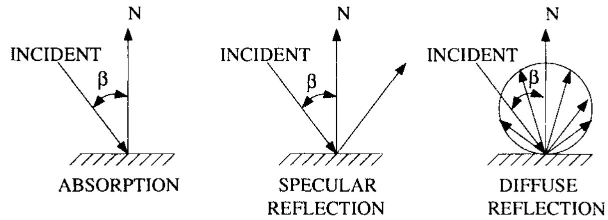

Fig. 3 Absorption and reflection of incident radiation.

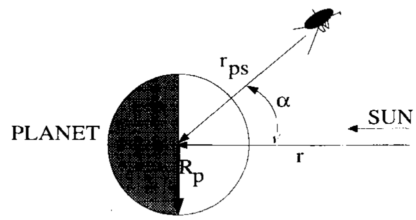

Fig. 4 Reflected solar radiation.

Solar Radiation

The magnitude of the force of solar radiation on most interplanetary spacecraft is second only to that of gravity. Thus, solar radiation is modeled in the trajectory analysis of the Galileo spacecraft (and so far this is the only nongravitational force that is modeled). Equation (1) gives the magnitude, when f_{elm} is replaced by f_o / r_{ss}^2 where $f_o = 1353 \pm 21$ (W/m^2) is the solar constant[4] that specifies the mean solar flux at 1 AU, and r_{ss} is the spacecraft distance relative to the sun in units of AU.

Reflected Solar Radiation

A practical approach to modeling the intensity of reflected radiation from a planet is given by Blanco and McCuskey.[5] The energy flux received by a planet of radius R_p and distance r (in AU) from the sun is $\pi R_p^2 f_o / r^2$ (see Fig. 4). Let $P(0)$ be the reflected energy flux from the planet, received by an observer located at a distance r_{ps} and in the sun-planet line ($\alpha = 0$), and let $P(\alpha)$ be the reflected energy flux at the same distance r_{ps} but at an angle α. The phase function $\Phi(\alpha)$ is then defined by $\Phi(\alpha) = P(\alpha)/P(0)$, and $P(0)$ can be obtained by using the equation

$$2\pi r_{ps}^2 P(0) \int_0^{\pi} \Phi(\alpha) \sin(\alpha)\, \mathrm{d}(\alpha) = a \pi R_p^2 f_o / r^2$$

which expresses the fact that the reflected total integrated energy flux is equal to the total energy flux received times the Bond albedo a. For a diffusely reflecting sphere obeying Lambert's law, the phase function is $\Phi(\alpha) = [\sin(\alpha) + (\pi - \alpha)\cos(\alpha)]/\pi$, and the integral is found to be

$$\int_0^{\pi} \Phi(\alpha) \sin\alpha \, \mathrm{d}\alpha = 3/4$$

so that $P(0) = 2aR_p^2 f_o / (3r^2 r_{ps}^2)$. Thus, the magnitude of the force due to the reflected solar radiation is given by replacing

Table 1 Effective areas for the Galileo spacecraft, m^2

Force	Near Earth ($10R_E$)	Near Earth ($1.05R_E$)	Near Venus ($3.75R_V$)	Interplanetary (3 AU)	Near Jupiter ($4.0R_J$)[a]
Solar radiation and solar wind forces	13.20	13.20	13.20	14.16	13.36
Reflected solar radiation and planet thermal radiation	6.50	11.42	6.58	—	11.50
Meteoroid and newtonian drag forces	11.28	14.20	11.86	11.79	11.52 (11.32)[a]
Cosmic ray force	13.00	13.00	13.00	13.00	13.00

[a]Newtonian drag force is calculated at $5.3R_J$.

f_{elm} in Eq. (1) by $P(0)$:

$$F = 2k_{\text{elm}}AaR_p^2 f_o/(3cr^2 r_{ps}^2)$$

This gives the upper limit of the reflected radiation force, which occurs at $\alpha = 0$ where $\Phi(0) = 1$. The Bond albedo a is 0.39 for Earth,[6] 0.72 for Venus,[7] and 0.52 for Jupiter.[7]

Planet Thermal Radiation

The average power E_o emitted by a planet with surface temperature T is determined by the Stefan-Boltzmann law

$$E_o = \sigma\xi T^4 \tag{2}$$

where

E_o = emissive power, W/m^2
ξ = nondirectional emissivity (dimensionless)
σ = Stefan-Boltzmann constant, 5.67×10^{-8} W/[m^2(K)^4]
T = surface temperature, K

The total emitted power P is given by $P = 4\pi E_o R_p^2$. Assuming that the planet acts as a blackbody, a spacecraft at a distance r_{ps} from the planet will receive a power per square meter $E(r_{ps})$ approximated by $E_o R_p^2/r_{ps}^2$. Thus, the force [Eq. (1)] due to thermal radiation is determined by

$$F = k_{\text{elm}}AE_oR_p^2/cr_{ps}^2 \tag{3}$$

According to Harris and Lyle,[2] the diffusely emitted thermal radiation of the Earth is approximated by a 255 K blackbody ($\xi = 1$). The Jupiter blackbody temperature is found to be 134 K (Newburn and Gulkis[7]). The effective blackbody temperature of Venus is approximately 240 K (Wertz[8]).

Spacecraft Radiation

The thermal radiation parallel to the spin axis (the z axis) of the Galileo spacecraft is analyzed as follows. In the $-z$ direction, the thermal radiation from the antenna and the sun shield is taken into account through the solar radiation model by modifying the coefficient k_{elm} in Eq. (1). The total average thermal power radiated in the $+z$ direction is approximately 50 W with an uncertainty of $3\sigma = 25$ W. The force resulting from this radiation is given by

$$F = P/c \tag{4}$$

where P is the emitted power (W) of the spacecraft.

The maximum power due to emitted radio signals from the antenna is 30 W; and the resulting force is also calculated by using Eq. (4). All radiation forces perpendicular to the spin axis of the spacecraft are assumed to cancel out over a spin period.

Particle Collision Forces

During its flight, the spacecraft will be hit by particles (such as meteoroids or nucleons) that impose an impulsive force on the spacecraft from each collision. To calculate an average continuously acting force, it is assumed that the spacecraft surface can be described by an effective plate area, which is hit by particles with a relative velocity V_R with respect to the spacecraft (see Fig. 5). The momentum change of the spacecraft in the case that the particles stick after the collision is approximately $\Delta p \cong mV_R$ with the assumption that $m_{\text{spacecraft}} \gg m$.

The average force due to collisions with particles of mass m and relative velocity V_R is now given by

$$F = \frac{\mathrm{d}p}{\mathrm{d}t} \cong \Delta p \frac{\mathrm{d}n}{\mathrm{d}t}$$

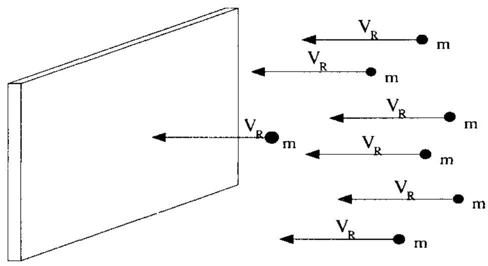

Fig. 5 Collision force model.

where $\mathrm{d}n/\mathrm{d}t = \rho AV_R/m$ is the number of particles per second and ρ is the density of the particle clouds. The disturbing force is then given by

$$F = \rho AV_R^2 \tag{5}$$

The term ρV_R (kg/m^{2}s) can be interpreted as mass flux rate. If the particle velocity is very large compared to the spacecraft, then V_R is approximately equal to V_{Particle} (this is true for solar wind particles and cosmic rays). The term ρV_R^2 (kg/ms^2) can then be regarded as the kinetic energy density or the momentum flux rate of the particle clouds.

Solar Wind

The momentum flux of the quiet solar wind at 1 AU in the ecliptic plane is given by[8] $\rho V^2 = 2.3 \times 10^{-9}$ kg/ms^2. In Eq. (5), ρV_R^2 is then replaced by $\rho V^2/r_{ss}^2$ where r_{ss} is the sun-spacecraft distance in units of AU.

Meteoroids

Near Earth Meteoroid Environment at 10R_E and 1.05R_E

Cour-Palais[9] developed a total meteoroid flux mass model that combines sporadic and stream meteoroids but assumes only cometary meteoroids. The asteroidal component is neglected. The average total cumulative meteoroid flux at 1 AU is given by the following expression:

$$\log_{10}N(m) = a + b\log_{10}m + c(\log_{10}m)^2 \tag{6}$$

where

$N(m) = Am^{b+c\log m}$ and $A = 10^a$
$N(m)$ = number of particles with mass m or greater per square meter per second
a,b,c = model constants for a defined mass range, $m_e < m < m_o$

For the purpose of a force calculation, it is necessary to convert $N(m)$ into a mass flux rate ρV_R.

Let $\Delta m = (m_o - m_e)/k$ (where k is an integer), and let m_i be the center of an ith interval. Then the mass flux rate can be expressed by

$$\rho V = \lim_{k\to\infty} \sum_{i=1}^{k} (\rho V)_i \tag{7}$$

where

$(\rho V)_i$ = mass flux rate of particles with mass m_i
$= m_i[N(m_i - \Delta m/2) - N(m_i + \Delta m/2)]$

Considering the limits the flux rate is

$$\rho V = \int_{m_e}^{m_o} m \frac{-\mathrm{d}N}{\mathrm{d}m}\,\mathrm{d}m \tag{8}$$

In the case where the quadratic term in Eq. (6) vanishes ($c = 0$), the mass flux rate can be calculated analytically by setting $dN(m)/dm = Abm^{b-1}$, and evaluating the integral

$$\rho V = -\int_{m_e}^{m_o} Abm^b \, dm = Ab \frac{(m_e^{b+1} - m_o^{b+1})}{(b+1)} \quad (9)$$

If $c \neq 0$, which is the case for the very small mass range of the meteoroid models, the integration can be performed numerically using Eqs. (6–8).

According to Cour-Palais,[9] the model constants are as follows: for $10^{-12} < m < 10^{-6}$ (m in grams), $a = -14.339$, $b = -1.584$, and $c = -0.063$; and for $10^{-6} < m < 10^{0}$, $a = -14.37$ and $b = -1.213$.

The numerical evaluation of Eqs. (7) and (8) with Eqs. (6) and (9) gives a total cumulative mass flux rate of $\rho V = 6.13 \times 10^{-16}$ kg/m²s. Note that this value does not include any body-shielding effects or lunar-ejected particles. So far, the velocity V included in the mass flux rate ρV describes only the relative velocity of the meteoroids with respect to the Earth. To calculate the force in Eq. (5), first the density ρ is calculated by dividing ρV by the average meteoroid velocity ($V_c = 20$ km/s at the entry of Earth's atmosphere) and then is multiplied by V_R^2 where V_R is the relative velocity between meteoroids and the spacecraft. The spacecraft departure velocity with respect to the Earth is $V_s = 5.28$ km/s at $10R_E$ and $V_s = 14.1$ km/s at $1.05R_E$, while the meteoroids have an average velocity of $V_c = G \times 20$ km/s directed towards the Earth. Therefore the maximum relative velocity is given as $V_R = V_s + V_c = 17.5$ km/s and 33.7 km/s for $10R_E$ and $1.05R_E$, respectively, where $G = 0.61, 0.98$ are the defocussing factors that correct for the Earth's gravity enhancement effect at $10R_E$ and $1.05R_E$, respectively.

Venus Meteoroid Environment at $3.75R_V$

To model the meteoroid environment of Venus, the method developed by D. J. Kessler[10] for interplanetary and planetary meteoroid environments is used to determine the density and average relative velocity of the meteoroids. Kessler's planetary meteoroid environment model is a cometary meteoroid model based on the Cour-Palais meteoroid flux model at 1 AU, with corrections for radial variations and heliocentric latitude.

Using this model for the Venus flyby at 0.72 AU ($3.75R_V$) and at a heliocentric latitude of 3.32 deg, the meteoroid density and average relative velocity are $\rho = 1.446 \times 10^{-19}$ kg/m³ and $V = 9769$ m/s.

Interplanetary Meteoroid Environment at 3 AU

During an interplanetary spacecraft flight, meteoroids of cometary as well as asteroidal origin must be considered.

The cometary meteoroid density and average relative velocity for cometary meteoroids at 3 AU and a heliocentric latitude of 1.04 deg from the Kessler[10] model are $\rho_c = 1.84 \times 10^{-20}$ kg/m³ and $V_c = 21.53 \times 10^3$ m/s. Kessler[10] also developed a model for the asteroidal meteoroid environment that gives an asteroidal meteoroid density and average relative velocity of $\rho_a = 2.44 \times 10^{-18}$ kg/m³ and $V_a = 17.98 \times 10^3$ m/s.

Jupiter Meteoroid Environment at $4R_J$

To model the meteoroid environment at Jupiter, the model of Kessler was modified according to the experimental results of Pioneers 10 and 11 by Humes[11] and Beck.[12] The changes assume no asteroidal component and that the spacial density is twice that given by Kessler at 1 AU, with no variation with radial distance or heliocentric latitude. Using the modified Kessler model, we find that the total integrated meteoroid density and average relative velocity for particle masses within the range $10^{-12} < m < 10^2$ (m in grams) are $\rho = 1.98 \times 10^{-19}$ kg/m³ and $V = 42.98 \times 10^3$ m/s.

Cosmic Rays

According to Divine,[13] the galactic cosmic rays have a typical energy density (for the entire population) of $\rho V^2 = 10^{-13}$ J/m³, which leads to a negligible force of 10^{-12} N for the Galileo spacecraft (since the surface area is roughly 10 m²). This is six orders of magnitude below Galileo mission requirements.

Newtonian Drag

Earth

From Allen,[6] the density of the Earth's atmosphere at 50,000 km altitude is $\rho = 2.5 \times 10^{-19}$ kg/m³. This value is used to estimate the upper limit of the Newtonian drag force at $10R_E$. The density of the Earth's atmosphere at $1.05R_E$ is reported by Wertz[8] to be 2.4×10^{-11} kg/m³. The velocity of the Galileo spacecraft relative to the Earth's atmosphere at $10R_E$ is 5.28 km/s and at $1.05R_E$ is 14.1 km/s.

Venus

Barth[14] states that the outer atmosphere of Venus is composed of molecular and atomic hydrogen and that the density at an altitude of 5948 km is 2.66×10^{-15} kg/m³. Extrapolating this value out to $3.75R_V$ using the exponential atmosphere method developed by Vinh et al.[15] results in a density of 4.25×10^{-18} kg/m³. The relative velocity of the Galileo spacecraft at this distance from Venus is 8.16 km/s.

Interplanetary

Wertz[8] reports that the proton density of the quiet solar wind at 1 AU is 8.7×10^6 protons/m³, resulting in a density of 1.5×10^{-20} kg/m³. We assume that the density at 3 AU is approximately the same value. The velocity of the Galileo spacecraft at 3 AU is 17.6 km/s. Preliminary results from the Galileo indicate that the proton density at 1 AU is one proton/cm³ (about an order of magnitude smaller). To be conservative, the value given by Wertz will be used in the force calculations.

Jupiter

The Newtonian drag effect near Jupiter at $4R_J$ is primarily due to the atmosphere of Io (which has a semimajor axis of $5.95R_J$). Data for the atmospheric density is found in Bagenal and Sullivan,[16] where the composition of the plasma in the dayside magnetosphere of Jupiter is reported from Voyager measurements for various distances. By summing the results for the individual ions (hydrogen, oxygen, sodium, and sulfur) we obtain a density of $\rho = 7.13 \times 10^{-17}$ kg/m³ at $5.3R_J$. Because this is the highest density that the Galileo passes through in its trajectory, $5.3R_J$ will be used for the Newtonian drag calculations. The velocity of the Galileo spacecraft relative to the Jovian atmosphere at $5.3R_J$ is 26.5 km/s.

Magnetic Field Forces

The interactions between the spacecraft with a charge q and an environmental magnetic field $\boldsymbol{B}$ is described by the Lorentz law:

$$F = q \,|\boldsymbol{V}_R \times \boldsymbol{B}| \quad (10)$$

where q is the spacecraft charge in coulombs (C), $\boldsymbol{V}_R$ is the relative velocity (m/s), and $\boldsymbol{B}$ is the magnetic induction in teslas (T).

The calculation of the magnitude of the relative velocity $|\boldsymbol{V}_R|$ is based on the assumption that the magnetic fields rotate with the same frequency as the originating body and that the spacecraft velocity vector is in the magnetic equatorial plane. Then the magnitude of the relative velocity is given by $|\boldsymbol{V}_R| = |V_{s/c} \mp (2\pi r_{ps}/T)|$, where T is the rotational period and the plus sign corresponds to flybys in which the magnetic field moves in a direction opposite to the spacecraft velocity. Taking into account the flyby geometry and the retrograde rotation of Venus, the minus sign is used for the launch from

Earth, the Venus flyby, the interplanetary cruise and the arrival at Jupiter, whereas the plus sign corresponds to the two Earth flybys.

Nonpropulsive Mass Expulsion Force

The resulting force due to pressurized gas leakage in the propulsion system can be calculated by using the rocket equation:

$$F = p_e A_e + \frac{dm}{dt} V_e \tag{11}$$

where

p_e = exit pressure at the leak, N/m^2
A_e = leak surface area, m^2
dm/dt = gas mass flow rate, kg/s
V_e = exit velocity, m/s

The upper limit of the leakage rate, dm/dt, is provided by spacecraft design tests. Although the nominal operating temperature and pressure are known, nothing is known about the area through which the pressurized gas leaks, nor the pressure over this area so that Eq. (11) cannot be applied directly. The following development circumvents this problem by providing an equation that gives the mass expulsion force in terms of the mass flow rate, stagnation temperature, and other known parameters. Using the one-dimensional continuity equation for the mass flow rate, $dm/dt = \rho AV$, we obtain

$$F = p_e A_e + \rho A_e V_e^2$$

Assuming a perfect gas yields[18] $\rho V^2 = kpM^2$, where k is the ratio of specific heat, p is the pressure (N/m^2), and M is the Mach number. Therefore, F can be written as

$$F = p_e A_e (1 + kM^2)$$

For an isentropic flow, the mass flow rate divided by the area is given by[18]

$$\left(\frac{dm}{dt}\right) A = pM(k/RT_o)^{1/2}\left[1 + (k-1)/2M^2\right]^{1/2}$$

where R is the gas constant for the pressurization gas [J/(kg-K)] and T_o the stagnation temperature (K), which is assumed to be identical with the pressurized tank temperature.

Assuming sonic conditions at the exit ($M = 1$), we obtain

$$F = \frac{dm}{dt}\left[2RT_o(1+k)/k\right]^{1/2} \tag{12}$$

It is interesting to note that this equation can be used to assign a specific impulse to a gas.

Torque Models

A detailed survey of spacecraft torques can be found in the study by Wiggins.[19] For the sake of simplicity, we assume that the torques can be approximated as constants. There are three kinds of torques to be considered:

1) Spin axis torques that effect a spin rate change. They may originate in inertial space or in the body. The spin rate change is given by $\Delta\omega = T_z \Delta t / I_z$.

2) Precessional torques that cause spin axis drift. These torques are normal to the spin axis and are inertially fixed. They cause the spin axis and the angular moment vector to drift by the angle $\tan^{-1}(\Delta H/H) = (T_x^2 + T_y^2)^{1/2} \Delta t / I_z \omega$.

3) Body-fixed torques normal to the spin axis that have no secular effect. These torques have a bounded effect on the perturbation of the spin axis and the angular momentum vector where the average value is given by the angle[20] $\tan^{-1}(\Delta H/H) = (T_x^2 + T_y^2)/I_z \omega^2$.

The bounded effect of the third kind of torque is quite small compared to that of the first two and, therefore, will be neglected in the following discussion. All of the forces mentioned so far apply a precessional and/or a spin axis torque. An additional torque is produced by the gravity gradient.

Torque Model for the Electromagnetic Radiation, Particle Collision, and Mass Expulsion Forces

The torques from electromagnetic radiation, particle collision, and mass expulsion forces can be calculated by

$$\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F} \tag{13}$$

where $\boldsymbol{r}$ is the torque arm (m) and $\boldsymbol{F}$ is the force (N).

Center of Pressure Offset

A precessional torque results from the offset of the c.p. and the c.m. Both electromagnetic radiation and particle collision forces produce effects that are calculated from Eq. (13). In the case of the Galileo spacecraft, the average torque arm is 0.445 m.

Propeller Effect

Spin axis torque on a spacecraft also occurs due to the propeller effect. This torque arises when a spacecraft boom is oriented such that, when modeled as a flat plate, the plane containing the boom is not precisely perpendicular to the spin axis. The misalignment causes a torque due to components of the electromagnetic radiation and particle collision forces acting on the boom's effective surface area parallel to the spin axis. This torque is calculated using Eq. (13), where F is the force component perpendicular to the spin axis and r is the distance from the c.m. of the boom to the spin axis of the spacecraft.

The booms considered in the torque calculations for the Galileo are the magnetometer science boom (area = 4.09 m^2, $r = 5.98$ m)[21], (see Ref. 21) and the two RTG booms (area = 0.85 m^2, $r = 3.78$ m)[21], (see Ref. 21) which are assumed to have misalignments of approximately 10 mrad each.

Spin Rate Decay

In the case of particle collision forces, spin rate decay is possible. Wiggins[19] has shown that an upper bound on the aerodynamic spin decay torque for a cylinder is roughly given by

$$T = -(F\omega r^2 / V_R)\left[\tfrac{1}{2}\cos(\theta)\cos(\psi) + \sin(\theta)\sin(\psi)\right] \tag{14}$$

where

F = particle collision force (N), Eq. (5)
ω = spin rate, s^{-1}
r = radius of the cylinder (m) (for Galileo $r \cong 1.0$ m)
V_R = relative velocity between spacecraft c.m. and the atmosphere
ψ = angle between the spin axis and relative velocity, V_R
θ = angle between the spin axis and the particle force, F

The Galileo spacecraft attitude angles, ψ and θ are given in Table 2.

Mass Expulsion

Mass expulsion forces cannot create a secular precession torque, but may decrease or increase the spin rate.

Torque Model for the Magnetic Field Forces

There are four sources of magnetic disturbance torques: 1) permanent magnetism in the spacecraft, 2) spacecraft generated current loops, 3) magnetism induced by an external field, and 4) currents induced by an external field. The first

Table 2 Orientation angles for the Galileo spacecraft, deg

Torque	Angle	Near Earth ($10R_E$)	Near Earth ($1.05R_E$)	Near Venus ($3.75R_V$)	Interplanetary (3 AU)	Near Jupiter ($4.0R_J$)[a]
Solar radiation and solar wind [Eqs. (13) and (14)]	Ψ	77	55	31	59	72
	θ	0[b]	0[b]	0[b]	12	2
Reflected solar and planet thermal radiation [Eq. (13)]	θ	87	45	86	12	43
Meteoroid and newtonian drag [Eqs. (13) and (14)]	$\Psi = \theta$	77	55	31	59	72 (76)[a]
Gravity gradient [Eq. (19)]	β	87	45	86	12	43

[a] Newtonian drag torque is calculated at $5.3R_J$.
[b] Maximum attitude excursion allowed by the HGA is 20 mrad.[25]

two torques can be described by

$$T = M \times B \tag{15}$$

where M is the spacecraft magnetic moment (A-m^2) and B is the ambient magnetic flux density (T). The Galileo magnetic moment is approximately 2.0 ± 1.2 (A-m^2), but the orientation of the moment is unknown. For an upper limit, it is assumed that the orientation is parallel to the spin axis which then leads to an effective precessional torque. Torques created by dipole components perpendicular to the spin axis are assumed to cancel out. Torques created by induced magnetism are negligible due to the usage of impermeable materials in the Galileo spacecraft.

The torque due to the interaction of induced currents (eddy currents) with an external field can be described by the following:[22]

Despin component:

$$T_p = k_e (B_o)^2 \omega_s \tag{16}$$

Precession component:

$$T_o = k_e \omega_s B_p B_o \tag{17}$$

where

B_o = component of the ambient magnetic field orthogonal to the spin axis, T
B_p = component of B parallel to the spin axis, T
ω_s = spin angular velocity (rad/s) = $(3.15 \times 2\pi/60)$ s^{-1}
k_e = constant (m$^4/\Omega$) which depends on the geometry and conductivity of the spinning surface.[22] For a thin-walled cylinder with length L, radius r, thickness d, and conductivity σ, $k_e = \pi\sigma r^3 Ld[1 - 2d/L \tanh(L/2d)]$

In the torque calculation, the Galileo is approximated by a thin-walled cylinder with $L = 0.46$ m, $d = 0.0038$ m, $r = 0.78$ m, and $\sigma = 3.8 \times 10^7\ \Omega^{-1}$ (beryllium) to represent the spacecraft bus structure. The amount and type of material in the high-gain antenna, the magnetometer science boom, the radioisotope thermoelectric generators, and other parts do not contribute significantly in this calculation. The magnitudes of the magnetic fields are given in Table 3.

Table 3 Magnetic field force data

Spacecraft position	B, T	T, s	$V_{s/c}$, km/s
Near Earth[17]			
$10R_E$	3.0×10^{-8}	8.64×10^4	5.28
$1.05R_E$	2.6×10^{-5}	8.64×10^4	14.1
Near Venus[17]			
$3.75R_V$	6.6×10^{-10}	2.11×10^7	8.16
Interplanetary[13]			
3 AU	2.0×10^{-9}	2.16×10^6	17.6
Near Jupiter[7]			
$4R_J$	6.3×10^{-6}	3.25×10^4	30.3

Table 4 Galileo spacecraft related parameters

Parameter	Value	Units	Equation
$k_{\rm elm}$	1.5 ± 0.23	—	1, 3
q	1.0×10^{-8}	C	10
dm/dt	5.4×10^{-10}	kg/s	12
R	2.077×10^3	J/kg-K	12
T_o	300	K	12
k	1.667	—	12
$(r_{\rm spin})$[a]	1.30	m	13
M	2.0 ± 1.2	A-m^2	15
k_e	9.739×10^4	m$^4/\Omega$	16, 17
I_z	5.00×10^3	kg-m^2	19
$I_x \cong I_y$	3.44×10^3	kg-m^2	19

[a] Mass expulsion spin axis torque.

Gravity Gradient Torque

Gravity gradient torque is due to the variation of the gravitational force over the distributed mass of the spacecraft. Detailed discussions of this torque are provided by Wiggins[19] and by Harris and Lyle.[23] Assuming a central inverse square field, the gravity gradient torque is given by

$$T_G = (3G/R^5)(R \times I \cdot R) \tag{18}$$

where G is the gravitational constant of the planet (m^3/s^2), R is the radius vector from the attracting planet to the spacecraft c.m. (m), and I is the moment-of-inertia tensor of the spacecraft (kg m^2). Using the coordinate system of Fig. 6, Eq. (18) provides the following inertial components:

$$T_X = (3G/R^3)\left[I_z - (I_x \sin^2\alpha + I_y \cos^2\alpha)\right] \sin\beta \cos\beta$$

$$T_Y = (3G/R^3)(I_x - I_y) \sin\beta \cos\beta \sin\alpha \cos\alpha$$

$$T_Z = (3G/R^3)(I_y - I_x) \sin^2\beta \sin\alpha \cos\alpha$$

where

X,Y,Z = inertially fixed axes
x,y,z = body-fixed axes where the z axis is the spin axis
I_x, I_y, I_z = principal moments of inertia
R = radius vector from planet to spacecraft c.m. (R lies in the Y-Z plane)

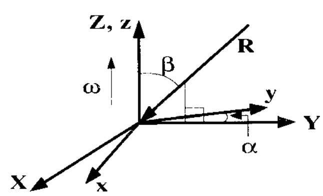

Fig. 6 Coordinate reference system.

β = angle between the spin axis and $\boldsymbol{R}$
α = angle between the Y axis and the y axis

Because of the fact that $\alpha = \omega t$, the torque components T_Y and T_Z cancel out over one complete spin revolution. $T_Z = 0$ implies that gravity gradient torques cannot cause spin rate changes.

The effective torque averaged over one spin revolution is given by

$$T_X = (3G/R^3)\left[I_z - \tfrac{1}{2}(I_x + I_y)\right] \sin\beta \cos\beta \tag{19}$$

Table 3 lists the angles β used to calculate the average effective gravity gradient torques.

Probabilistic Error Modeling

Probabilistic error models can be obtained from the governing equations for electromagnetic radiation forces, particle collision forces, magnetic field forces, nonpropulsive mass expulsion force, and the torque models. Since the method is standard practice, it will not be discussed here. A detailed treatment is given by Longuski and König.[24]

Table 5 Mean and 3σ of environmental forces on the Galileo spacecraft, N

Source	Near Earth (launch, $10R_E$)		Near Earth (flyby 2, $1.05R_E$)		Near Venus ($3.75R_V$)		Interplanetary (3 AU)		Near Jupiter ($4.0R_J$)[a]	
	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ
1. Electromagnetic radiation										
Solar radiation	9.0×10^{-5}	2.6×10^{-5}	9.2×10^{-5}	2.6×10^{-5}	1.7×10^{-4}	4.9×10^{-5}	1.1×10^{-5}	3.0×10^{-6}	3.3×10^{-6}	9.3×10^{-7}
Reflected solar radiation	1.2×10^{-7}	3.7×10^{-8}	1.9×10^{-5}	6.0×10^{-6}	2.9×10^{-6}	9.4×10^{-7}	—	—	6.1×10^{-8}	1.9×10^{-8}
Planet thermal radiation	7.8×10^{-8}	5.2×10^{-8}	1.2×10^{-5}	8.2×10^{-6}	4.4×10^{-7}	2.9×10^{-7}	—	—	6.6×10^{-8}	4.4×10^{-8}
Spacecraft radiation	2.7×10^{-7}	4.0×10^{-8}	2.7×10^{-7}	4.0×10^{-8}	2.7×10^{-7}	4.0×10^{-8}	2.7×10^{-7}	4.0×10^{-8}	2.7×10^{-7}	4.0×10^{-8}
2. Particle collision force										
Solar wind	3.1×10^{-8}	1.6×10^{-8}	3.1×10^{-8}	1.6×10^{-8}	5.9×10^{-8}	3.0×10^{-8}	3.6×10^{-9}	1.8×10^{-9}	1.1×10^{-9}	5.6×10^{-10}
Meteoroids	1.1×10^{-10}	7.4×10^{-10}	4.9×10^{-10}	3.4×10^{-9}	1.6×10^{-10}	1.2×10^{-9}	9.4×10^{-9}	2.6×10^{-7}	4.2×10^{-9}	3.0×10^{-8}
Cosmic rays	0[b]	1.3×10^{-12}	0[b]	1.3×10^{-12}	0[b]	1.3×10^{-12}	0[b]	1.3×10^{-12}	0[b]	1.3×10^{-12}
Newtonian drag	7.9×10^{-11}	2.4×10^{-10}	6.8×10^{-2}	2.1×10^{-1}	3.4×10^{-9}	2.5×10^{-8}	5.3×10^{-11}	4.2×10^{-10}	5.7×10^{-7}	1.7×10^{-6}
3. Magnetic field force	1.9×10^{-13}	3.1×10^{-13}	3.5×10^{-9}	5.7×10^{-9}	5.4×10^{-14}	8.8×10^{-14}	2.1×10^{-11}	2.0×10^{-10}	1.6×10^{-9}	2.5×10^{-9}
4. Nonpropulsive mass expulsion force	7.6×10^{-7}	2.5×10^{-7}	7.6×10^{-7}	2.5×10^{-7}	7.6×10^{-7}	2.5×10^{-7}	7.6×10^{-7}	2.5×10^{-7}	7.6×10^{-7}	2.5×10^{-7}
Mission requirement (Mean + 3σ)	7.7×10^{-6}		7.4×10^{-6}		6.9×10^{-6}		6.6×10^{-6}		4.1×10^{-6}	

[a] Newtonian drag force is calculated at $5.3R_J$.
[b] The mean energy density for cosmic rays is zero.

Table 6 Mean and 3σ of secular precessional torques on the Galileo spacecraft, N-m

Source	Near Earth (launch, $10R_E$)		Near Earth (flyby 2, $1.05R_E$)		Near Venus ($3.75R_V$)		Interplanetary (3 AU)		Near Jupiter ($4.0R_J$)[a]	
	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ
1. Electromagnetic radiation										
Solar radiation	0[b]	3.2×10^{-7}	0[b]	3.2×10^{-7}	0[b]	6.1×10^{-7}	1.0×10^{-6}	4.0×10^{-7}	4.0×10^{-8}	1.6×10^{-8}
Reflected solar radiation	5.1×10^{-8}	2.2×10^{-8}	5.9×10^{-6}	2.5×10^{-6}	1.3×10^{-6}	5.5×10^{-7}	—	—	1.8×10^{-8}	7.8×10^{-9}
Planet thermal radiation	3.5×10^{-8}	2.5×10^{-8}	3.9×10^{-6}	2.8×10^{-6}	2.0×10^{-7}	1.4×10^{-7}	—	—	2.0×10^{-8}	1.4×10^{-8}
Spacecraft radiation	1.2×10^{-7}	4.0×10^{-8}	1.2×10^{-7}	4.0×10^{-8}	1.2×10^{-7}	4.0×10^{-8}	1.2×10^{-7}	4.0×10^{-8}	1.2×10^{-7}	4.0×10^{-8}
2. Particle collision torques										
Solar wind	0[b]	2.8×10^{-10}	0[b]	2.9×10^{-10}	0[b]	5.4×10^{-10}	3.4×10^{-10}	3.5×10^{-10}	1.4×10^{-11}	1.4×10^{-11}
Meteoroids	4.8×10^{-11}	3.4×10^{-10}	1.8×10^{-10}	1.3×10^{-9}	3.7×10^{-11}	2.6×10^{-10}	3.6×10^{-9}	3.3×10^{-8}	1.8×10^{-9}	1.3×10^{-8}
Cosmic rays	0[c]	5.8×10^{-13}	0[c]	5.8×10^{-13}	0[c]	5.8×10^{-13}	0[c]	5.8×10^{-13}	0[c]	5.8×10^{-13}
Newtonian drag	3.4×10^{-11}	1.1×10^{-10}	2.5×10^{-2}	7.8×10^{-2}	7.8×10^{-10}	5.9×10^{-9}	2.0×10^{-11}	1.7×10^{-10}	2.5×10^{-7}	7.5×10^{-7}
3. Magnetic field torques										
Magnetic dipole moment	6.0×10^{-8}	2.3×10^{-8}	5.2×10^{-5}	2.0×10^{-5}	1.3×10^{-9}	1.2×10^{-9}	3.3×10^{-9}	9.0×10^{-8}	1.2×10^{-5}	1.1×10^{-5}
Eddy current	9.3×10^{-12}	1.8×10^{-11}	6.9×10^{-6}	1.3×10^{-5}	7.1×10^{-15}	2.2×10^{-14}	4.5×10^{-14}	8.0×10^{-13}	2.1×10^{-7}	6.9×10^{-7}
4. Gravity gradient torque	3.2×10^{-7}	3.9×10^{-8}	3.1×10^{-3}	8.4×10^{-5}	8.4×10^{-6}	7.1×10^{-7}	1.4×10^{-12}	5.1×10^{-14}	1.3×10^{-5}	7.6×10^{-7}

[a] Newtonian drag is calculated at $5.3R_J$.
[b] Maximum attitude excursion allowed by the HGA is 20 mrad (mean is zero).
[c] The mean energy density for cosmic rays is zero.

Table 7 Mean and 3σ of secular spin axis torques on the Galileo spacecraft, N-m

	Near Earth (launch, $10R_E$)		Near Earth (flyby 2, $1.05R_E$)		Near Venus ($3.75R_V$)		Interplanetary (3 AU)		Near Jupiter ($4.0R_J$)[a]	
Source	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ	Mean	3σ
1. Eddy current	2.6×10^{-11}	4.8×10^{-11}	1.9×10^{-5}	3.6×10^{-5}	7.1×10^{-15}	2.2×10^{-14}	4.5×10^{-14}	8.0×10^{-13}	1.2×10^{-6}	3.7×10^{-6}
2. Mass expulsion	9.9×10^{-7}	3.0×10^{-6}	9.9×10^{-7}	3.0×10^{-6}	9.9×10^{-7}	3.0×10^{-6}	9.9×10^{-7}	3.0×10^{-6}	9.9×10^{-7}	3.0×10^{-6}
3. Propeller effect	0[b]	4.1×10^{-8}	0[b]	9.8×10^{-4}	0[b]	1.9×10^{-7}	0[b]	4.0×10^{-8}	0[b]	3.0×10^{-9}
4. Particle collision										
Solar wind	3.8×10^{-15}	8.0×10^{-15}	9.9×10^{-15}	2.1×10^{-14}	2.8×10^{-14}	5.8×10^{-14}	2.9×10^{-16}	6.0×10^{-16}	1.5×10^{-16}	3.2×10^{-16}
Meteoroids	2.0×10^{-15}	1.4×10^{-14}	4.0×10^{-15}	2.9×10^{-14}	3.4×10^{-15}	2.4×10^{-14}	1.5×10^{-13}	4.2×10^{-12}	3.1×10^{-14}	2.2×10^{-13}
Newtonian drag	4.8×10^{-15}	1.6×10^{-14}	1.3×10^{-6}	4.2×10^{-6}	8.7×10^{-14}	6.6×10^{-13}	8.6×10^{-16}	7.0×10^{-15}	6.9×10^{-12}	2.3×10^{-11}

[a] Newtonian drag is calculated at $5.3R_J$.
[b] Propeller effect is the result of misalignment of spacecraft components; therefore, mean torque is zero.

Numerical Results for the Galileo Mission

The analytical models can now be applied to the Galileo spacecraft to obtain mean and 3σ values for nongravitational forces and environmental torques. The spacecraft related parameters used in the calculations of the forces and torques are listed in Tables 1–4.

For the environmental forces, Table 5 clearly shows that the solar radiation pressure dominates during most of the trajectory. In fact, this force and its uncertainty exceed the mission requirements for all but the Jovian portion of the mission. For this reason, the solar radiation force on the Galileo spacecraft is modeled in the trajectory analysis. In practice, it is difficult to reduce the uncertainties of the model parameters to the requirement level, therefore inflight calibrations are currently being performed. The mission requirements are also exceeded for reflected solar radiation, planet thermal radiation, and Newtonian drag forces on the second Earth flyby and for reflected solar radiation near Venus. These forces are presently being studied by the Galileo mission operations teams. Since the forces are short-lived, fairly simple models should suffice in the analysis. The remaining environmental forces vary in magnitude below the mission requirements.

The numerical results for the precessional torques on the Galileo spacecraft are presented in Table 6. The largest effects are encountered on Galileo's second flyby of Earth, and are a result of Newtonian drag and gravity gradient torques. The Newtonian drag torque is greater than 2×10^{-2} N-m. The next largest torques, on the order of 10^{-5} N-m, occur at Jupiter, and are due to the magnetic dipole moment and gravity gradient torques.

The spin axis torques are reported in Table 7. All of the torques given in this table are despin torques except for the mass expulsion and propeller effect torques, which may be either spin up or despin torques. The predominant effect is that of the propeller torque on the second Earth flyby, which is 9.8×10^{-4} N-m (3σ).

Further details on the numerical results for the Galileo mission are provided in Refs. 24 and 26.

Conclusions

In the application to the Galileo spacecraft, the models indicate that solar radiation effects must be accounted for in the trajectory analysis in order to satisfy mission requirements. This was no surprise—all recent interplanetary spacecraft trajectories are modeled to include gravitational fields and solar radiation forces. It was also expected that the short-lived effect of Newtonian drag during the Galileo's second Earth flyby would temporarily exceed the requirements. What is surprising is the fact that effects that were thought to be negligible, such as reflected solar and planet thermal radiation, nonpropulsive mass expulsion, and spacecraft radiation, are now within (or near) one order of magnitude of current mission requirements. In future missions, some of these effects may have to be modeled in the trajectory analysis along with the gravitational and solar radiation forces. To achieve more stringent requirements than those of the Galileo mission, better understanding of the nature of these forces will be necessary.

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